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Theory of Interaction and Structure Formation in Liquid Crystal Colloids and Emulsion

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Theory of Interaction and Structure Formation in Liquid Crystal Colloids and Emulsion

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This article is semi review our result about interaction and structure formation in liquid crystal colloids and emulsion. We treat colloidal and emulsion systems in liquid crystal using effectively describing the elastic interaction. Here we consider a system whose interactions are presumably well-understood and the phase transition in which can be studied in the scale of macro particle resolution – the model system allowing to study the mechanisms of the structural phase transitions. We presented analytical results of possible structures for suspension of macro particles in liquid crystal. We have shown that cellular structure formation can occur in liquid crystals for realistic values of temperature and macroparticle concentration.

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I. INTRODUCTION

At the same time physical properties (optical, electrical, magnetic and rheological) of the anisotropic melt substantially vary at doping liquid crystals. Depending on geometrical dimensions and on the dopant effect, one can distinguish between two types of those – the micro- and

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macro-dopes. Micro-dopes are inclusions having molecular dimensions, and with supplementary physical properties being different from the anisotropic melt molecules properties. Such inclusions can locally change the surrounding; this results in change of global characteristics of anisotropic melt. Macro dopes, because of large enough surface, depending on the value of adhesion to that surface, prescribe boundary conditions for the liquid crystal molecules orientation; this leads to local deformation of the director elastic field. Two types of adhesion to the macro particles surface are distinguished between – the strong anchoring and the weak anchoring. At strong anchoring, the surface itself prescribes the director orientation, and the surface itself acts as the source of the director elastic field change. For the weak field, each inclusion acts as disturbance of orientation of the director, because of the fact that the boundary conditions at the surface do not correspond to the real distribution of the director at the area of the separate inclusion location only. It is obvious that macro-inclusions of arbitrary size immersed to liquid crystal lead to deformation of equilibrium distribution of the director and the area of that deformation, depends on the value of the adhesion. It is shown, for instance, that the director distribution round a spherical inclusion at normal boundary conditions on its surface has a topological defect existing as a hyperbolic urchin, or dislocation loop [9]. At tangent boundary conditions on the surface of the inclusion, two budjums come to existence, and distribution of the director takes spindle-like shape [1,2]. Thus, it is obvious that any inclusion of macroscopic dimensions leads to disturbance of the director distribution. In the case of the area of disturbance being large enough, at overlapping of the director elastic field deformation areas for two and more macro-inclusions there can be formed effective interaction between those. Effectiveness of such interaction shall be determined, first of all, by geometrical parameters of the inclusion, force of adhesion of the liquid crystal molecules to the surface of such an inclusion and elastic properties of medium. In general case, value and character of interaction are determined by the value and character of violation of the director distribution symmetry [13].

In the case of strong anchoring, surface of separate inclusion completely determines orientation of the director round it, and the value of the area of deformation of the director [3–10]. Taking into account the director distribution round separate inclusions, one can find the change of orientation of the director induced by two macro-inclusions, and determine the change of deformation energy at the approach of those with separation of the energy component corresponding to interaction between those. Besides that, the liquid crystal itself is a nonlinear medium, from the definition of the director itself, since its module is always equal to unity, and none of linear operations, like superposition of separate

disturbances of the director, can be applied in general case. Reconstructions of the real distribution of the director at approach of two macro-inclusions by means of linear operations of superposition seem to be rather ambiguous. Though, in such an approach, and at weak bonding, one can determine far-range character of interaction depending on configuration of possible distribution of the director at small distances round a specific macro-inclusion. In such approach, character of far-range interaction is determined by the character of the symmetry disturbance for the short-range director distribution only.

In self-consistency approach only it is possible to determine collective effect by macro-particles [1]. Result of collective interaction, with overlapping of deformation areas, is screening of this interaction. This relates, first of all, to colloids and emulsions with high concentration of inclusions. Such screening is anisotropic in its character, and it is absent for spherical particles without topological charges in director distribution round specific macro-inclusion.

For the first time, possible structures in the system of macro-inclusions in liquid crystal at weak bonding, those that can take place in such a system, were predicted in the article [1]. Existence of elastic interaction is the cause of formation of cellular structures in the system of small spherical particles with weak bonding observed by Anderson as well [15,16] and the case feromagnetic particle in liquid crystal [11]. It is found that elastic interaction through the director field leads to spatially inhomogeneous distribution of such particles with formation of regions of pure liquid crystal. In those regions there shall exist topological defects in the director distribution. Minimum of free energy realizes cellular structure and leads to stabilization of defects in the regions of pure liquid crystal, and the particles themselves are placed at the boundaries of those regions. To describe the conditions of formation and properties of these macro particle structures, one should take into account both aspects of the particle interaction. The interaction energy associated with the deformation of the director field was conducted as a function of distance and physical characters. At first we shall give representation of the general approach in the description of behavior in system of cooperating particles. Let's show as in this system can be realized of a phase first order with creation of spatially non-uniform distribution of particles.

II. SYMMETRY AND THE DIRECTOR DISTRIBUTION

A nematic liquid crystal is an anisotropic fluid, in which long molecules have the same average orientation, specified by the unit vector

n, called the director. In the undistorted state the nematic has a spatially uniform orientation $\mathbf{n}_0$, and we consider here that it is parallel to the axis z ($\mathbf{n}_0 = (0,0,1)$). The case of global nonuniform director distribution is investigated in the article [13], where the expression for the energy of an arbitrary particle in the curved director field is found.

Immersed particles distort the uniform orientation of the director in the bulk. The *source* of bulk director deformations is the preferential orientation imposed at the surface of particles in such a way that the nematic molecules lie either normally or tangentially to it. The phenomenological anchoring energy at the surface of particles can be written in the Rapini-Papoular form

$$F_s = \sum_p W \oint ds(\nu(\mathrm{s}) \cdot \mathbf{n}(\mathrm{s}))^2, \tag{1}$$

where W is the anchoring energy coefficient. Bulk energy of the spatial distortions of the director field, that is called the Frank energy, is written in the form

$$F_b = \frac{1}{2}\int d^3r\{K_{11}(div\,\mathbf{n})^2 + K_{22}(\mathbf{n}rot\mathbf{n})^2 + K_{33}(\mathbf{n}\times rot\mathbf{n})^2\}, \tag{2}$$

In order to find possible director configurations one should solve Euler-Lagrange (EL) equations from the minimization of the Frank free energy with taking into account of the boundary conditions, which are found from the minimization of bulk and surface energies. But the situation can arise, when there are several director distributions with different symmetry, which satisfy both EL equations in the bulk and boundary conditions at the surface. The uniform director distribution far from the droplet has zero topological charge and so there should be another topological defect near the droplet to compensate the hedgehog in the center. Obviously, the director configurations have different symmetry. The non equatorial disclination ring and the pair of radial and hyperbolic hedgehogs break mirror symmetry in the horizontal plane, while equatorial disclination ring (Saturn-ring) retains it. Saturn-ring configuration has quadrupole symmetry, which is reduced to the dipolar symmetry, when the disclination ring is shifted above or below the equator. Authors of [18] have shown by Monte-Carlo simulations, that the configuration with hyperbolic hedgehog has lower energy, than Saturn-ring. It has been confirmed in [9] with help of the dipole Ansatz, that though the equatorial ring has some metastability, its energy is higher, than of the dipole.

So, on this example, we see, that strong anchoring on the surface breaks mirror symmetry, though the shape of the particle remains spherical. For weak anchoring or smaller particle's size (less than

1 μm) it is not, and the quadrupole symmetry configuration remains [19], just as the droplet itself has.

The breaking of the symmetry in the near-field region that is achieved either by the anchoring strength or by the particle's shape, leads to the different solutions in the far-field region. At the far distances from the particle, the director field $\mathbf{n}(\mathbf{r})$ tends to be uniform $\mathbf{n}_0 = (0, 0, 1)$ and can be written in the form $\mathbf{n} = (n_x, n_y, 1)$. In the one-constant approximation the Frank free energy is given by

$$F_b = \frac{1}{2} K \int d^3 r \{(\nabla n_x)^2 + (\nabla n_y)^2\} \tag{3}$$

Equilibrium equations are the Laplace equations for the transverse components n_μ $(\mu = x, y)$

$$\Delta n_\mu = 0 \tag{4}$$

At large distances r it can be expanded in multipoles

$$n_\mu = \frac{A\mu}{r} + \frac{\mathbf{P}_\mu \mathbf{r}}{r^3} + \frac{c_\mu^{ij} r_i r_j}{r^5} + \cdots \tag{5}$$

The first term is connected with the "charge", the second with the dipole moment and the last term is connected with the quadrupole moment.

Three items represent different broken symmetries of the director field around particles and are responsible for three different interaction law between particles, as we show below. The first term exists, when the director distribution does not have any plane of the symmetry at all or it has only one vertical plane of the symmetry. It appears when the particle in it's vicinity breaks mirror symmetry in a horizontal plane and in one vertical plane.

The multipole expansion is valid only in the region, where nonlinearities can be neglected. For particles with strong anchoring it is the far-region, because of strong director deformations in the near-region. But for particles with weak anchoring distortions are small elsewhere and the multipole expansion is applicable in the near-region too. In general the less anchoring strength, the less size of the region where the multipole expansion is inapplicable.

III. MIRROR SYMMETRY BREAKING AND THE DIPOLE–DIPOLE INTERACTION

In this chapter we intend to clarify the appearance of the dipole moment $\mathbf{p}$ with breaking of the mirror symmetry of the director field

in one plane in the vicinity of the surface and to represent $\mathbf{p}$ as the measure of the skewness.

In the paper [9] dipolar and quadrupole moments are expressed as $\mathbf{p}_\mu = (\mathbf{p}\cdot\mathbf{n}_0)\mathrm{e}_\mu$ and $c_\mu^{ij} = c(n_{0i}e_j^\mu + n_{0j}e_i^\mu)$ where e_μ are the vectors pointing in the $\mu = x, y$ direction. The authors have found the interaction potential in the far region between the water droplets with hyperbolic hedgehogs, when the dipolar moments lies parallel to the director. Their result is found in the one constant approximation ($K_{11} = K_{22} = K_{33} = K$) and it is written

$$U(\mathbf{R}) = 4\pi K\left[p_z p_z' V_{pp}(\mathbf{R}) + \frac{4}{9}cc' V_{cc}(\mathbf{R}) + \frac{2}{3}(cp' - c'p)V_{pc}(\mathbf{R})\right] \qquad (6)$$

$$V_{pp}(\mathbf{R}) = \frac{1}{R^3}(1 - 3\cos^2(\theta))$$

$$V_{cc}(\mathbf{R}) = \frac{1}{R^5}(9 - 90\cos^2(\theta) + 105\cos^4(\theta))$$

$$V_{pc}(\mathbf{R}) = \frac{\cos(\theta)}{R^4}(15\cos^2(\theta) - 9)$$

where θ is the angle between the separation vector $\mathbf{R}$ and $\mathbf{n}_0$; p_z, p_z', c, c' are dipolar and quadrupole moments respectively at the positions $\mathbf{r}$ and $\mathbf{r}'$. From the variational *Ansatz* it has been found, that $p_z = 2.04a^2$, $c = -1.08a^3$, a being the droplet's radius. The last term in (6) is absent for equal droplets, and only the dipole–dipole $V_{pp}(\mathbf{R})$ and the quadrupole–quadrupole $\mathrm{V_{cc}}$ $(\mathbf{R})$ potentials remain.

In the paper [1] authors suggested the approach, which enables to find the interaction potential for particles of ordinary shape with weak anchoring at the surface. It is valid for the different Frank constants and so exceed the bounds of the electrostatic analogy. In the weak anchoring case there are no topological defects and the director deformations $\delta\mathbf{n}$ are small everywhere, so that the multipole expansion is valid at the particle surface too.

In the case of strong anchoring topological defects arise in the near-region, but outside the director deformations $\delta\mathbf{n}$ are small. Therefore we can confine the particle and topological defects within the region that is called coat. This region contains all strong deformations of the director field. Outside the coat director deformations are small $\delta\mathbf{n} \ll 1$. The size of the coat has been estimated in [13] with help of the electrostatic analogy and it was shown to be few micrometers from the particle's surface. It is in a qualitative agreement with the result of [9] $l = 0.26R_0$ for the distance between the droplet's surface and the

hyperbolic hedgehog, when the radius R_0 of the droplet is about ten micrometers. The symmetry of the coat is equivalent to the broken symmetry of the director in the vicinity. For example, a droplet with equatorial disclination ring (Saturn-ring configuration) could be put into the coat which has a horizontal symmetry plane and a water droplet with a companion hyperbolic hedgehog could be confined into the coat without a horizontal symmetry plane. The anchoring energy on the surface of the coat is determined as the interaction energy between the nematic molecules over the surface and the molecules under the surface of the coat. Phenomenologically it can be written in the Rapini-Papoular form, but it is natural that the anchoring strength W_c becomes dependent on the point s on the surface of the coat. The shape and the surface distribution $W_c(s)$ determine the symmetry of the coat which is identical to the symmetry of the real director distribution in the near-region. Obviously, it is necessary to know the real director field in every point to find $W_c(s)$ and shape of the coat exactly, that is hardly achieved. Instead of finding exact solutions of the EL equations, we show that the problem can be effectively solved in terms of some unknown tensors which characterize the surface of the coat. For this purpose we introduce the surface energy in the form

$$F_{cs} = \sum_p \oint ds W_c(s)(\nu(s) \cdot \mathbf{n}(s))^2 \tag{7}$$

Such substitution allows us consider the director deformations $\delta\mathbf{n}$ small everywhere in the space. All defects are hidden now inside the coat.

This makes it possible to find the interaction potential between the coats. We can apply the results of the paper [1], because $\delta\mathbf{n} \ll 1$ far and wide. It was the only physical assumption under which the results are valid.

According to the [1] interaction potential between the two coats separated by the vector $\mathbf{R}$ in general case is written as:

$$\begin{aligned} U(\mathbf{R}) = -\frac{1}{8\pi} \sum_{m,m'=1,2,3} \hat{A}^p_m \hat{A}^{p'}_{m'} \sum_{\mu=1,2} \frac{1}{\sqrt{K_{\mu\mu}}} \\ \times \left\{ \frac{Q^+_{m,m'}}{\sqrt{K_{33}R_\perp^2 + K_{\mu\mu}R_\parallel^2}} + (-1)^\mu \frac{Q^-_{m,m'}}{R_\perp^2} \right. \\ \left. \times \frac{\left(\sqrt{K_{33}R_\perp^2 + K_{\mu\mu}R_\parallel^2} - \sqrt{K_{\mu\mu}R_\parallel^2}\right)^2}{\sqrt{K_{33}R_\perp^2 + K_{\mu\mu}R_\parallel^2}} \right\} \end{aligned} \tag{8}$$

In this expression $R_{||}$ and $R_{\perp}$ are parallel and perpendicular to the undeformed director $\mathbf{n}_0$ components of the $\mathbf{R} = r_p - r_{p'}$:

$$\mathbf{r}_1 = \frac{\mathbf{R}_{\perp} \times \mathbf{n}_0}{R_{\perp}}; \quad \mathbf{r}_2 = \frac{\mathbf{R}_{\perp}}{R_{\perp}}; \quad r_3 = \mathbf{n}_0; \quad \mathbf{R}_{\perp} = \mathbf{n}_0 \times \mathbf{R} \tag{9}$$

$Q^{(\pm)}_{m,m'} = (\mathrm{r}_1 \cdot \mathrm{k}_m)(\mathrm{r}_1 \cdot \mathrm{k}_{m'}) \pm (\mathrm{r}_2 \cdot \mathrm{k}_m)(\mathrm{r}_2 \cdot \mathrm{k}_{m'})$, where $(\mathrm{k}_1, \mathrm{k}_2, \mathrm{k}_3)$ is the local basis rigidly bound with each particle. Operators $\hat{A}_m$ are defined as:

$$\hat{A}_m = (\mathrm{k}_l \mathbf{n}_0)[\alpha_{lm} + \beta_{lms}(\mathbf{k}_s \cdot \nabla) + \gamma_{lmst}(\mathbf{k}_s \cdot \nabla)(\mathbf{k}_t \cdot \nabla)] \tag{10}$$

Superscript p means that in operator $\hat{A}^p_m$ we need to substitute $\nabla = \partial/\partial \mathbf{r}_p$.

Here $\alpha_{lm}, \beta_{lms}, \gamma_{lmst}$ are tensor characteristics of the coat, which contain all information about its symmetry. Let it be ρ the vector pointed from the center of mass of the particle to the point s at the surface of the coat and ν is the unit normal to the surface at this point. Then tensors are expressed as

$$\begin{aligned} \alpha_{kl} &= 2 \oint d\sigma W_c(\mathrm{s}) \nu_k(\mathrm{s}) \nu_l(\mathrm{s}) \\ \beta_{kl} &= 2 \oint d\sigma W_c(\mathrm{s}) \nu_k(\mathrm{s}) \nu_l(\mathrm{s}) \rho_m(\mathrm{s}) \\ \gamma_{klmn} &= \oint d\sigma W_c(\mathrm{s}) \nu_k(\mathrm{s}) \nu_l(\mathrm{s}) \rho_m(\mathrm{s}) \rho_n(\mathrm{s}) \end{aligned} \tag{11}$$

The integration is over the surface of the coat. The symmetry of these tensors contains all data about broken symmetry of the director field in the vicinity of particles and defines distinctive features of the interaction potential in the far-region. The magnitudes of it can be treated like variational parameters for the concordance with experimental data or it can be evaluated from the comparison with long-range asymptotic of *Ansatzes* solutions.

$$U(\mathrm{R}) = -\frac{1}{8\pi} \sum_{m,m'=1,2,3} \hat{A}^p_m \hat{A}^{p'}_{m'} \left(\frac{Q^+_{m,m'}}{R} \right) \tag{12}$$

It can be easily shown, that $Q^+_{m,m'} = 0$ for m or m' equal 3, and $Q^+_{m,m'} = \delta_{m,m'}$, for $m, m' = 1, 2$. Using of this allows us to write the expression in the form

$$\begin{aligned} U(\mathrm{R}) = &-\frac{\alpha_{3m}\alpha_{3m}}{4\pi R} + \beta_{3ms}\beta_{3ms'}(\mathrm{k}_s \cdot \nabla)(\mathrm{k}_{s'} \cdot \nabla)\frac{1}{4\pi R} \\ &- \gamma_{3mst}\gamma_{3mst}(\mathrm{k}_s \cdot \nabla)(\mathrm{k}_{s'} \cdot \nabla)(\mathrm{k}_t \cdot \nabla)(\mathrm{k}_{t'} \cdot \nabla)\frac{1}{4\pi R} \end{aligned} \tag{13}$$

where the summation on the repeating indices is made.

Now we consider the case of $\alpha_{3m} = 0$ and below we show under what circumstances this is not true. Now we want to examine thoroughly the second term, which represents the dipolar–dipolar interaction. It takes the form

$$U_{dd} = \beta_{3ms}\beta_{3ms'}\frac{(3(\mathrm{k}_s \cdot \mathrm{R})(\mathrm{k}_{s'} \cdot \mathrm{R}) - \delta_{ss'})}{R^3} \tag{14}$$

Symmetry of the tensor β reflects symmetry of the director and defines dipolar type of the interaction. Let us consider components $\beta_{31s} = 2\oint d\sigma W_c(\mathrm{s})\nu_1\nu_3\rho_\mathrm{s}$. If the director distribution has horizontal symmetry plane, then for every point ν exists the mirror image, for which ν_3 changes the sign whereas ν_1, ρ_1, ρ_2 remain the same. Therefore $\beta_{311} = \beta_{312} = 0$. Besides, if there is symmetry plane YZ, then $\beta_{313} = 0$, else $\beta_{313} \neq 0$. Similar, if there is symmetry plane XZ, then $\beta_{323} = 0$, else $\beta_{323} \neq 0$. For example, banana-shaped particles with the symmetry plane XZ have $\beta_{313} \neq 0$, and thus induce elastic dipolar–dipolar interaction as the result of the director distortions.

So we come to the *important conclusion*: if the director distribution near the particle has three perpendicular planes (one of which is horizontal), then all β_{3ms} are zero and only quadrupole interaction remains. If one symmetry plane in the director distribution is broken, then a dipolar moment in the perpendicular direction arises.

If the horizontal symmetry plane is broken, as in the case of the water droplets in nematics [4], then nonzero components of the tensor β are $\beta_{311} = \beta_{322}$, and dipolar part takes the form:

$$U_{dd} = \frac{\beta_{311}^2}{4\pi K R^3}(1 - 3\cos^2(\theta)) \tag{15}$$

with $\cos\theta = \mathrm{n}_0\mathrm{R}/R$. This coincides with the first term, and we clearly see that β_{311} is proportional to the z component of the dipolar moment p_z:

$$\beta_{311} = -4\pi K p_z$$

(we choose "-" in order to agree the results on the behavior of particles in the curved director field with [9]). To demonstrate the appearance of the dipolar moment as the result of the breaking of the mirror symmetry, we consider a model system – a sphere with nonequatorial thread, on which anchoring strength is infinite. Surface anchoring on the sphere can be written as

$$W_c(\theta) = W_0 + W_1(\cos(\theta)) \cdot \delta(\cos(\theta) - \cos(\theta_{th}))$$

W_0 is the constant anchoring on the surface and they second term is the anchoring on the thread. Then integration on the sphere surface gives

$$\beta_{311} = 2\oint d\sigma W_c(\mathrm{s})\nu_1\nu_3\rho_1 = 2\pi \mathrm{R}^3 \mathrm{W}_1(x_{th})\cdot x_{th}\cdot(1-x_{th}^2)$$

where $x_{th} = \cos(\theta_{th})$. Inasmuch as $W_1(0) \neq 0$, then we conclude, that

$$\beta_{311} \sim \cos(\theta_{th})$$

when $\theta_{th} \approx \pi/2$. It becomes zero, when the thread lies on the equator and changes its sign, when the thread is shifted above or below the equator, so that it really behaves similar to the dipolar moment.

When the mirror symmetry is not broken, then all components of the tensor β are zero and the last term remains, which represent quadrupole–quadrupole interaction. In this case nonzero components of the tensor γ which are included in the interaction are γ_{3131}, and γ_{3232}. In general they are different, if the particle do not have axis of symmetry C_4. In this case the general quadrupole–quadrupole interaction potential is

$$\begin{aligned} U_{qq} = &-\frac{3\gamma_{3131}^2}{2\pi KR^5}(1-5\cos^2\theta - 5(\mathrm{k}_1\mathrm{e}_r)^2 + 35\cos^2\theta(\mathrm{k}_1\mathrm{e}_r)^2) \\ &-\frac{3\gamma_{3232}^2}{2\pi KR^5}(1-5\cos^2\theta - 5(\mathrm{k}_2\mathrm{e}_r)^2 + 35\cos^2\theta(\mathrm{k}_2\mathrm{e}_r)^2) \end{aligned} \quad (16)$$

where $\mathrm{e}_r = \mathrm{R}/R$. We see, that potential depends not only on the angle θ but also on the azimuthal angle φ.

For the cylinders, which lie parallel to the axis Y $\gamma_{3232} = 0$. Then the interaction potential, for instance, for the cylinders, which lie in the horizontal plane ($\theta = \pi/2$) becomes

$$U_{qq\ horisont} = \frac{3\gamma_{3131}^2}{2\pi KR^5}(5\cos^2\varphi - 1)$$

$\cos\varphi = (\mathrm{k}_1\mathrm{e}_r)$, $\gamma_{3131} = \pi r^3 L W_c/4$. They repel in X direction and attract in Y direction.

If the coat has the axis of symmetry C_4 then γ_{3131}, and γ_{3232} are equal and the quadrupole interaction takes the form

$$U_{qq} = \frac{3\gamma_{3131}^2}{2\pi KR^5}(3 - 30\cos^2\theta + 35\cos^4\theta)$$

From the comparison with it we find to the sign the connection with the quadrupole moment c of the coat

$$\gamma_{1313} = c\frac{4\pi K}{3}\sqrt{2}$$

Then for particles with planar anchoring $W>0,\ \gamma_{1313}>0,\ c>0$ and for particles with homeotropic anchoring $\gamma_{1313}<0,\ c<0$ that is in an agreement with the *Ansatz* result for the quadrupole moment of the water droplet in nematic [9].

IV. THE POSSIBLE COULOMB-LIKE ELASTIC ATTRACTION

General expression shows that the Coulomb attraction is present in general case. Let us consider the nature of this term. It is determined by the components α_{13} and $\alpha_{23}, \alpha_{13} = 2\oint d\sigma W_c(\mathrm{s})\nu_1(\mathrm{s})\nu_3(\mathrm{s})$. It is obvious, that, if there is a horizontal symmetry plane, then both components are zero. The presence of the symmetry plane XZ makes $\alpha_{23}=0$ and YZ make $\alpha_{13}=0$. So we come to the *significant conclusion*: the breaking of both horizontal and vertical mirror symmetry leads to the $\alpha_{13}\neq 0,$ or $\alpha_{23}\neq 0$ and thus to the Coulomb attraction

$$U = -Q^2/R \tag{17}$$

where the role of the charge plays geometrical factor $Q = \sqrt{\frac{(\alpha_{13}^2+\alpha_{23}^2)}{4\pi K}}$.

We can calculate the charge for instance for the long cylinder ($L \gg d$), which makes the tilt angle θ with the z axis and lies in the YZ plane. For such cylinder $\alpha_{13}=0, \alpha_{23}\neq 0$. Let O' – the coordinate basis *(XY′Z′)*, which is turned by the angle $\frac{\pi}{2}-\theta$ with respect to the axis X. In this basis $\nu' = (\cos\varphi, 0, \sin\varphi)$. In the *(X, Y, Z)* basis $\nu_i = A_{ij}\nu'_j$, where the rotation matrix

$$A_{ij} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sin\theta & -\cos\theta \\ 0 & \cos\theta & \sin\theta \end{bmatrix}$$

so that $\nu_1 = \cos\varphi,\ \nu_2 = -\cos\theta\sin\varphi,\ \nu_3 = -\sin\theta\sin\varphi$. Then $\alpha_{23} = 2\oint d\sigma W\nu_2\nu_3 = -dL\pi W\cos\theta\sin\theta$ (we integrate only on the side area), and the charge is

$$Q = \frac{dL|W\sin 2\theta|}{4}\sqrt{\frac{\pi}{K}} \tag{18}$$

When the cylinder lies in the XY plane ($\theta=\frac{\pi}{2}$) or YZ plane ($\theta=0$) then $Q=0$, as follows from general symmetry considerations. The maximum charge occurs when $\theta=\pi/4$. Such inclined position of the cylinder is not of course profitable energetically. In the ground state the cylinders make angle $\theta=0$ or $\theta=\pi/2$ in dependence on the ratio Wd/K [34]. But, if they have magnetic moment, then the external

magnetic field can fix the tilt angle $\theta \neq 0, \pi/2$ and cause the elastic Coulomb attraction between them. Such situation is realized, for example, in the suspension of ferromagnetic particles in the nematic, which is called ferronematic [22], in the presence of the external magnetic field.

This Coulomb-like attraction can be found experimentally also between the liquid drops in the nematics in the presence of the inclined electric field. In the paper [25] it was shown, that isotropic drops suspended in the nematic phase (5CB and MBBA) are deformed into an elliptical shape along the field. If the direction of the electric field is neither parallel nor perpendicular to the director, than the drops should deform parallel to it, so that both horizontal and vertical mirror symmetry is broken. This must lead to the elastic Coulomb attraction between the drops.

V. SCREENING OF THE ELASTIC INTERACTION

In all previous papers concerning colloidal particles in liquid crystals, the elastic interaction potential between particles has been obtained as the result of the overlapping of the director distortions around the pair of particles. In principle, however, the rest of the particles should also have an influence on the interaction potential. In a previous paper [24], we have examined this case and showed that the deformations from all particles lead to the exponential screening of the pair interaction potential. Physically it can be explained following the argument by Brochard and de-Gennes [22].

Let us consider cylindrical particles, labelled by a unit vector u, and θ is the angle between n_0 and u. It produce director in the far region $\mathrm{n}(\mathrm{r}) = \mathrm{n}_0 + \delta\mathrm{n}(\mathrm{r})$, where $\delta\mathrm{n}(\mathrm{r}) = (\delta\mathrm{n}_x,\ \delta\mathrm{n}_y,\ 0)$. It is convenient to put $\delta\mathrm{n}(\mathrm{r}) = \omega(\mathrm{r}) \wedge \mathrm{n}_0$, where $\Delta\omega_x = \Delta\omega_y = 0$. Since ω_z is arbitrary, we may put $\Delta\omega_z = 0$. Then

$$\omega(\mathrm{r}) = \varkappa \frac{1}{r} + \chi : \nabla \frac{1}{r} + \cdots \tag{19}$$

We are interested mainly in the first item. In a nematic states n and $-\mathrm{n}$ are identical, therefore $\varkappa$ must be an even function of n_0. It must be also an even function of u. The most general vector constructed from u and n_0 and even in each of them is

$$\varkappa = l(\cos\theta)\mathrm{n}_0 \wedge \mathrm{u}$$

where $l(x)$ is an odd function of x. This leads us to

$$\delta\mathrm{n}(\mathrm{r}) = \frac{l(\cos\theta)}{r}\mathrm{u}_\perp$$

Note that the coefficient of $1/r$ vanishes when u is parallel to n_0 and also when u is normal to n_0 (since $l(0) = 0$). This coincides with our symmetry considerations made above. Let us define $l_1 = l(\cos\ \theta)$. In [22] it is shown, that the distortion energy is

$$\delta F = 4\pi K l_1 \sin\theta\, \delta\theta \tag{20}$$

from which it is seen, that if the parallel orientation of the particle satisfies the minimum energy, then l_1 should be positive.

If we have a suspension of identical particles with positions r_p, then the distortion $\delta n(r)$ from all of them is given by the following equation:

$$\delta n(r) = \sum_p \frac{l_1}{|r - r_p|}[u_{p\perp} - \delta n(r_p)]$$

The presence of $\delta n(r_p)$ means that the particle (p) creates no long deformations, if it is aligned parallel to the director $n_0 + \delta n(r_p)$.

Let us now go to the continuum approach, substituting $\Sigma \to c\int dV$, with c being the concentration of particles $c = N/V$. After acting with the operator ∇^2 on both sides

$$\delta n(r) = c l_1 \int dr' \frac{1}{|r - r'|} e^{-\xi_{exp}|r-r'|} u_\perp(r')$$
$$\xi^2_{exp} = 4\pi c l_1 > 0$$

Thus the effect of each particle is screened out at distances larger than ξ^{-1}_{exp}. It is obvious, that this screening should be manifested also in the interaction potential.

$$U_{pp'} = -\frac{1}{8\pi}\hat{A}^p_m \hat{A}^{p'}_{m'}[I^{exp}_{1ll'}(r_p - r_{p'}) + I^{exp}_{2ll'}(r_p - r_{p'})] \tag{21}$$

$$I^{exp}_{\mu m m'}(R) = \frac{1}{\sqrt{K_{\mu\mu}K_{33}}}[Q^+_{m,m'} + (-1)^{\mu+1}Q^-_{m,m'}]\frac{e^{-\xi_\mu\sqrt{\frac{K_{\mu\mu}}{K_{33}}R^2_\parallel + R^2_\perp}}}{\sqrt{\frac{K_{\mu\mu}}{K_{33}}R^2_\parallel + R^2_\perp}}$$
$$+ \frac{2(-1)^\mu}{\sqrt{K_{\mu\mu}K_{33}}}\frac{Q^-_{m,m'}}{\xi_\mu R^2_\perp}\left[e^{-\xi_\mu\sqrt{\frac{K_{\mu\mu}}{K_{33}}}R_\parallel} - e^{-\xi\mu\sqrt{\frac{K_{\mu\mu}}{K_{33}}R^2_\parallel + R^2_\perp}}\right] \tag{22}$$

$$\xi_\mu = \sqrt{\frac{c(a_{11} + a_{22})}{2K_{\mu\mu}}}$$

where ξ_μ are inverse screening lengths ($\mu = 1, 2$) and c is the concentration of particles; a_{11} and a_{22} are the corresponding components of

the tensor

$$a_{\mu\nu} = \alpha_{lm}[k_{l_\mu}k_{m_\nu} - k_{l_3}k_{m_3}\delta_{\mu\nu}]$$

In the one-constant approximation $K_{\mu\mu} = K_{33} = K$ it becomes dependent only on the scalar of the vector R:

$$U_{pp'} = -\frac{Q^{+}_{m,m'}}{8\pi K}\hat{A}^{p}_{m}\hat{A}^{p'}_{m'}\left[\frac{\exp(-\xi|\mathrm{r}_p - \mathrm{r}_{p'}|)}{|\mathrm{r}_p - \mathrm{r}_{p'}|}\right] \tag{23}$$

where $\xi = \sqrt{\frac{c(a_{11}+a_{22})}{2K}}$. It is clearly seen that the presence of the macroscopic concentration c of particles leads to the screening of the pair interaction potential with the screening length $\xi^{-1} \approx \sqrt{K/WcS}$ (we mean W is absolute value here, not depending on the sign), S-area of the particle. This screening takes place both for the homeotropic and for the planar anchoring. Concentration here is included in the inverse screening length ξ only, so that the limit $c \to 0$ makes $\xi = 0$ and gives us back to the unscreened result. The presence of the tensor $a_{\mu\nu}$ in the screening lengths indicates, that screening has anisotropic nature, i.e. it exists only for the anisotropic coats. For the small spherical particles (less than 1 μm) without any topological defects the coat coincides with the particle itself. In this case tensor $a_{\mu\nu}$ is identical to zero. Really $\alpha_{lm} = \alpha\delta_{lm}$ and $k_{l_\mu}k_{l_\nu} = \delta_{\mu\nu}$ so $a_{\mu\nu} \equiv 0$ for small spherical particles. This means, that no screening takes place. It arises only for the anisotropic coats. The physical meaning of this elastic screening can be easily understood from such example: two people badly hear one another in the crowded room. The more people between them the worse they hear one another. Similar with particles: is true, when the screening length is bigger than the average distance between particles $\xi^{-1} \gg \langle l\rangle = 1/\sqrt[3]{c}$. From here we can write the condition on the anchoring strength of the coat under which the given approach is applicable:

$$W_c \ll \frac{K}{\sqrt[3]{c}S_c} \tag{24}$$

VI. CELLULAR TEXTURE IN THE FERRONEMATICS

In 1970 Brochard and de Gennes proposed "doping" the liquid crystal matrix with ferromagnetic grains to make possible the coupling of the liquid crystal molecular orientation to weak external fields [22]. These authors treated such a system theoretically and made the prediction that the doped matrix should exhibit collective orientational distortion in the weak magnetic fields H $\sim 10G$. Also they predicted segregation effects i.e. the smooth change of the grains concentration c($\vec{R}$) from

point to point at applying of the magnetic field. In the paper [11] authors observed experimentally the collective behavior in the MBBA doped with magnetic grains which is exhibited as a long ranged uniform distortion of the molecular orientation of the entire sample upon application of the magnetic field as weak as <1 G. In that experiment the grains were coated with DMOAP, which provides homeotropic anchoring on its surfaces, so that the magnetic grains lie perpendicular to the nematic director in the absense of the magnetic field.

This system was studied theoretically by Burylov and Raikher [27,33]. It was shown, that under applying of the magnetic field H there is some angle between the dipole moment direction $\vec{m}$ (it is unit vector) of the grain and the director $\vec{n_0}$ which is different from $\pi/2$ or 0 for the finite anchoring.

In order to describe the experimental results on the dependence of the field induced birefringence on the strength of the applied field, on the concentration of the magnetic dopant and on the thickness of the nematic cell, Burylov and Raikher proposed such free energy density functional:

$$F = \frac{1}{2}[K_{11}(div\,\vec{n})^2 + K_{22}(\vec{n}\,rot\,\vec{n})^2 + K_{33}(\vec{n} \times rot\,\vec{n})^2] - M_s f \cdot (\vec{m}\,\vec{H}) + \frac{f k_b T \ln f}{v} - \frac{f W A (\vec{n}\vec{m})^2}{d} \tag{25}$$

Here f is the volume fraction occupied by the particles $f = cv$, v – is the particle volume; M_s is the magnetization inside the grains, d-diameter and $A \sim 1$ the constant. This functional differs from the one proposed by Broshard and de Gennes [22] only by the last term. The last term accounts for the weak anchoring under which $0 < \theta < \pi/2$. Minimization with respect to f (keeping the number of the particles fixed) leads to

$$f = f_0 \exp\left[\frac{\mu \vec{H}\,\vec{m}}{k_b T} + \frac{W A v (\vec{n}\,\vec{m})^2}{d k_b T}\right] \tag{26}$$

where f_0 is found from the total number of grains $\bar{f} = Nv = \int f(r) dV$. Burylov et al. have found that the particles accumulate in the centre of the cell under applying of the magnetic field. For the weak fields $H < 10$ G the dependence f(z) (z-the axis perpendicular to the cell, $z = 0$ in the centre) has the form

$$f(z) = \bar{f}[1 = \rho^2 D^2 (1 - 12 z^2/D^2)/48\lambda^2] \tag{27}$$

where $\lambda = \left(K_{33} v / 2\bar{f} k_b T\right)^{1/2}$, D – the thickness of the cell, $D \geq 100\,\mu$m, $\rho = M_s v H / k_b T$, $M_s \sim 340\,G$, $v \sim 2 \cdot 10^{-15}\,\text{cm}^3$. At higher fields

concentration is increased in the centre more faster that was found by computer simulation. But on reaching of the field $\mathrm{H} \sim 30\,G$ experiment shows [11] that the uniform orientational distortion is replaced by a new field-induced "cellular" texture with the "cells" having dimensions on the order of tens of micrometers. So at the critical concentration in the center magnetic particles clump into aggregates. This clumping had no explanation, because the magnetic dipole–dipole interaction is much smaller than the interaction with the external magnetic field. Indeed, the magnetic moment $\mu = M_s v$ induces interaction $E_{dd} = \mu^2/R^3$, where R is average distance between the particles $\mathrm{R}^{-3} \sim c \sim 10^{10}\,\mathrm{cm}^3$. So $E_{dd} \sim 4 \cdot 10^{-15}$erg. Energy of interaction with external magnetic field $\mathrm{H} \sim 10\,G$ is $E_H = \mu H \sim 3 \cdot 10^{-12}\,\mathrm{erg}$ and $E_H \gg E_{dd}$.

We explain this field induced "cellular" texture by the clumping of the grains which is caused by the elastic deformations of the director, i.e. by the elastic interaction between the particles. We use "+" because we think that the deviation of the θ from $\pi/2$ is small so that $\theta > \arcsin\sqrt{2/3}$. So

$$U_{pp'} = -\frac{1}{4\pi K}\hat{A}_l^p \hat{A}_{l'}^{p'} \circ Q_{l,l'}^{+} \frac{\exp(-\xi R)}{R} \tag{28}$$

In the operators $\hat{A}_l^p$ we leave only the first term $\hat{A}_l = \alpha_{lm}(\vec{n} \cdot \overrightarrow{k_m})$ because of others give the higher powers on $1/\mathrm{R}$. Here $Q_{l,l'}^{+} = (\overrightarrow{r_1} \cdot \overrightarrow{\kappa_l})(\overrightarrow{r_1} \cdot \overrightarrow{\kappa_{l'}}) + (\overrightarrow{r_2} \cdot \overrightarrow{\kappa_l})(\overrightarrow{r_2} \cdot \overrightarrow{\kappa_{l'}})$. For the cylinder the tensor $\alpha_{lm} = 2\oint ds W(s)\nu_l(s)\nu_m(s)$ has such components: $\alpha_{11} = \alpha_{22} = dL\pi W$, $\alpha_{33} = d^2\pi W$ others $\alpha_{lm} = 0$, L – the length of the grain. So $\alpha_{33}/\alpha_{11} = d/L \sim 0.1$ and we neglect α_{33}. Thus we obtain

$$U_{cyl}(R) = -\frac{\alpha_{11}^2 \sin^2\theta \cos^2\theta}{4\pi K}\frac{\exp(-\xi(\theta)R)}{R} \tag{29}$$

In the equilibrium orientations $\theta = 0, \pi/2$ screened Coulombic interaction is absent and the higher order terms remain. Let it be $e = \alpha_{11}^2 \sin^2\theta\cos^2\theta/4\pi K$. Now we want to consider the system of the particles with the concentration c and law interaction $U(r) = -e\frac{\exp(-\xi r)}{r}$. Free energy density of such a system is written in the form:

$$F = \frac{kT}{v}\int f(R)\ln f(R)dV + \frac{1}{2v^2}\int f(\vec{R})f(\vec{R}+\vec{r})U(r)d\vec{R}\,d\vec{r} \tag{30}$$

We need to find a condition of loosing stability in such system of the attracting particles. It means that the concentration becomes

$f(\vec{R}) = f_0 + \delta f(\vec{R})$, f_0 is the ground concentration. We make series expansion

$$f(\vec{R}+\vec{r}) \approx f(\vec{R}) + (\vec{r}\ \vec{\nabla})f(\vec{R}) + \frac{1}{2}(\vec{r}\ \vec{\nabla})^2 f(\vec{R})$$

Then we have:

$$F - F_0 = \frac{1}{2}\int N\delta f^2(\vec{R}) + M\left(\vec{\nabla}\delta f\right)^2 \tag{31}$$

$$N = 2kT/v + \frac{1}{v^2}\int_{r_0}^{\infty} U(r)d\,\vec{r}$$

$$M = -\frac{1}{2v^2}\int_{r_0}^{\infty} U(r)r^2 d\,\vec{r}$$

Here r_0 is the size of the particle. In as much as $\mathrm{U} < 0$ there phase transition occurs when $\mathrm{N} < 0$. In our case $\xi r_0 \ll 1$ and we may write

$$N \approx \frac{2kT}{f_0 v} - \frac{4\pi e}{\xi^2 v^2}$$

$$M \approx \frac{12\pi e}{\xi^4 v^2}$$

Below the critical point $N \sim 4\pi e/\xi^2 v^2$. The length of the first instability is

$$l_{inst} = \sqrt{2M/N} \sim \frac{1}{\xi} \tag{32}$$

Here $\xi^{-1}(\theta) = \sqrt{K/c|a(\theta)|}$ is the screening length. In the experiment of Chen $\xi^{-1} \sim 60\,\mu\mathrm{m}$ as we have found above.

For the concentration $c \sim 10^{10}\mathrm{cm}^{-3}$ the average distance between the particles is $\langle l\rangle \sim 5\,\mu\mathrm{m}$, so that $\xi^{-1} \gg \langle l\rangle$. There are about 1000 particles in the volume ξ^{-3} and they really lead to the screening. As we have discussed above $l_{inst} \sim 50\,\mu\mathrm{m}$ that is in a good agreement with the experimental size of the "cells" [11]. In the experiments of Chen and Amer [11] with the cylindrical particles the concentration is $\mathrm{c} \approx 10^{10}\,\mathrm{cm}^{-3}$, $\mathrm{S} \approx RL$. The radius of the grain is $\mathrm{R} \approx 0.05\,\mu\mathrm{m}$, the length $\mathrm{L} \approx 0.5\,\mu\mathrm{m}$. Elastic constant $\mathrm{K} \sim 10^{-7}\,\mathrm{dyn}$, anchoring energy $\mathrm{W} \sim 10^{-3}\,\mathrm{dyn/cm}$ and we find $\lambda_{res} \approx 60\,\mu\mathrm{m}$. The concentration and anchoring can be changed so that the resonance range is $\lambda_{res} \sim 1 - 100\,\mu\mathrm{m}$.

VII. CELLULAR STRUCTURE FORMATION IN LIQUID CRYSTAL COLLOIDS

To describe the peculiarities of the macro particle system behavior in the liquid crystal implies taking into account interaction via the director elastic field. We have already shown that a foreign macro particle produces liquid crystal distortion in a region much greater than the particle dimensions and thus leads to an effective interaction with another similar macro particle via the director field deformation. This macro particles may also be regarded as particle surrounded by a "solvate shell" provided the interaction between such a particle and the liquid crystal molecules is much more intense than the intermolecular interaction responsible for the liquid crystal formation. The solvate formation may be regarded as macro particle with size which is equal the size deformation coat around the particle, thus its interaction with another similar formation may be described in term of the director field deformation [14]. The validity of this treatment the spherical macro particles we can impose the boundary condition on the director for such a formation. In this sense, the interaction of the spherical macro particles also is associated with the director elastic field deformation in the liquid crystal in which these particles are dissolved. The problem of interaction between the particles of the nematic liquid crystals a solution in the self-consistent molecular field approximation for macro particles implies that the field distribution on the surface of an individual macro particle is determined by the joint effect of all other macro particles [13].

Having found the inter particle interaction energy, we can study the thermodynamic behavior of an aggregate of such macro particles and describe the condition for the creation of new structure. The character and intensity of the inter particle interaction in the system of foreign macro particles in nematic liquid crystal can be such that a temperature and concentration phase transition in the system and produce a spatially inhomogeneous distribution the macro particles. The is first-order face transition when the external field present itself the topological defect in nematic ordering. In the end this determined the scale of the inhomogeneous distribution of the macro particle in nematic liquid crystal. In order to demonstrate the mechanism and character of the phase transition accompanied by the formation of the inhomogeneous distribution of the system foreign spherical macro particles. In this case the free energy of solution of macro particles in liquid crystal in the self-consistent field and the many-body approximation may be written in the form:

$$F = F_p + F_s + F_n \tag{33}$$

where

$$F_p = \int U(\vec{r} - \vec{r'}) f(\vec{r}) f(\vec{r'}) d\vec{r}\ d\vec{r'} \tag{34}$$

is the free energy, in term function $f(\vec{r})$ spatially distribution macro particles in nematic liquid crystal to employ of the pair deformation energy interaction. The simplest free energy including a interaction terms that respect this local gauge symmetry.

$$F_s = \int \{f(\vec{r}) \ln f(\vec{r}) + [1 - f(\vec{r})] \ln[1 - f(\vec{r})] d\vec{r} \tag{35}$$

is the entropy part free energy. This kind the entropy part the free energy is motivation that the two classical particle not possible occupation own space place. The phenomenological free energy of deformation nematic liquid crystals [22]

$$F_n = \int \{K_{11}(div\ \vec{n})^2 + K_{22}(\vec{n} \cdot rot\, \vec{n})^2 + K_{33}(\vec{n} \times rot\, \vec{n})^2\} d\vec{r} \tag{36}$$

where K_{ii} are the elastic frank constants, and $\vec{n}$ is the director, describe the elastic deformation product by all foreign macro particles. In ours case we have the topological defects, which are in ordered media singular regions of spatial dimensions less than that of physical space that are surrounded by order-parameter configuration that cannot be transformed to a homogeneous ground state via continuous deformation. The free energy of deformation nematic may be rewrite in follow form:

$$F_d = \int \{1 - f(\vec{r})\} \varepsilon(R) d\vec{r} \tag{37}$$

where $f(\vec{r})$ is the distribution function spherical region of the radius with topological defect, $\varepsilon(R)$ is the density energy deformation liquid crystal.

The minima of the free energy corresponds to the self-consistent field solution for $f(r)$. Each of thermodynamic functions of state corresponds to a solution that describes some phase of macro particle arrangement. If their distribution can be inhomogeneous, then the solution serves to find the stable phase associated with the interaction temperature and character. The minima of the free energy corresponds to the self-consistent field solution for $f(r)$. Each of thermodynamic functions of state corresponds to a solution that describes some phase of macro particle arrangement. If their distribution can be inhomogeneous, then the solution serves to find the stable phase associated with the interaction temperature and character. If the

macro particle solution is disordered, then by definition the mean value $f(\vec{r}) = c$, where c is the relative macro particle concentration. The concentration inhomogeneity gives rise to an additional term $f(\vec{r}) = c \pm \varphi(\vec{r})$ where $\varphi(\vec{r})$ is the change of the probability distribution function of the macro particles. If the concentration inhomogeneities are smooth and their scale is much longer the inter particle distance, the quantity may be interpreted as the change of macro particle composition. When passing from to continuum description, we can write the free energy increment, associated with the inhomogeneous macro particles distribution in the term of the power series expansion in using the long-wavelength expansion of the concentration i.e. $\varphi(\vec{r'}) = \varphi(\vec{r}) + \vec{\rho}_i \partial_i \varphi(\vec{r}) + \frac{1}{2}\vec{\rho}_i \vec{\rho}_j \partial_j \partial_i \varphi(\vec{r}) +$. Where $\vec{\rho} = \vec{r} - \vec{r'}$ distance between two particles. In this case we may be rewrite the part free energy, which is dependence from the change of the probability distribution function of the macro particles in the form:

$$\Delta F(\varphi) = \int d\vec{r} \left\{ \frac{1}{2} l^2 (\nabla \varphi)^2 - \frac{1}{2}\mu^2 \varphi^2 + \frac{1}{4}\lambda \varphi^4 - m \in \varphi \right\} \tag{38}$$

where

$$\mu^2 \equiv \left(V - \frac{kT}{c(1-c)} \right), \quad V = \int U(\vec{\rho}) d\vec{\rho}, \quad l^2 = \int U(\vec{\rho}) \vec{\rho}^2 d\vec{\rho}, \tag{39}$$

λ is determined in the term many-body interaction energy. The coefficient responsible for nonlinearity of system, which is induced the many-body interaction in system macro particles foreign in liquid crystal. In this case not exist none even terms because the function distribution macro particles satisfy the relation: $\int f(\vec{r}) d\vec{r} = N$, $\int \varphi(\vec{r}) d\vec{r} = 0$. The expression is the Landau free energy of a system macro particles which foreign in nematic liquid crystal, below the phase transition temperature of this system. Thus we see that the minimum of the functional realizes a spatially inhomogeneous macro particle distribution only provided the sing satisfy some relation and the values of coefficients determined by the inter particle interaction. In order to reveal the condition under which the homogeneous macro particles distribution become unstable, we have to calculate all the coefficient. Temperature of the phase transition to new states the inhomogeneous distribution of particles may be determined from following relation;

$$kT_c = c(1-c)V \tag{40}$$

The functional is very good know functional, which description the first-order phase transition with accompanied the cluster formation in the system of the interacting particles. Thus, the description in term of

the concentration which serves for the order parameter. The most important contribution in the concentration is associated with the field configuration for which the value of the free energy is minimum, i.e.:

$$\Delta\varphi - \frac{dV}{d\varphi} = 0$$

where $V = -\frac{1}{2}\mu^2\varphi^2 + \frac{1}{4}\lambda\varphi^4 - m \in \varphi$ is potential energy in our case. Substitution the solution in the expression from the free energy yield and its variation due to the formation of new phase. In the isotropic case when the difference of minimum effective potential values is greater than the barrier height, the solution in our case the free energy one cluster is described by the expression:

$$F = 4\pi \int_0^\infty r^2 dr \left\{ \frac{1}{2}\left(\frac{d\varphi}{dr}\right)^2 + V(\varphi) \right\} = -\frac{4\pi}{3} r^3 \varepsilon + 4\pi r^2 \sigma \qquad (41)$$

where σ is the surface energy of the cluster boundary that is equal to the free energy corresponding to the solution of the one-dimensional problem, i.e.

$$\sigma = \int_0^\infty dr \left\{ \frac{1}{2}\left(\frac{d\varphi}{dr}\right)^2 + V(\varphi) \right\} = \int_0^\infty d\varphi \sqrt{2V(\varphi)} \qquad (42)$$

where the integral should be calculated from external field. The radius of the new phase cluster by the minimum of the free energy. It is given by $R_0 = 2\sigma/\varepsilon$. In the our case we have $\varepsilon = 2\mu\epsilon/\lambda^{\frac{1}{2}}$ and $\sigma = \mu^3/3\lambda$, then $R_0 = \mu^2 l/3\lambda^{\frac{1}{2}}\epsilon$ in our case, and the effective value of the free energy variation from cluster formation is given by

$$F = m\left\{ (1-c)\varepsilon(R) + \frac{4\pi\sigma^3\lambda}{3\mu^2 m^2 \varepsilon^2(R)} \right\} \qquad (43)$$

In the case we assume that the external field all topological defects, which we calculate from homogeneous distribution macro particles are slow than order parameter variation-region of deformation director, we have possibility determined the number disclinasion in liquid crystal. To this number, we have to find its condensate value from minimum of the free energy: $\delta F/\delta\varphi = 0$. Thus we obtain in 3D case:

$$m\varepsilon(R) = \left\{ \frac{8\pi\mu^7}{27\lambda(1-c)} \right\}^{1/3} \qquad (44)$$

They sometimes possess mixed physical properties of their elements, but in many cases quite new properties emerge, reflecting new structural organization of their elements. The characteristic

dimensions of the spatially in-homogeneous distribution of the concentration macro particles in the end, become a criterion the first order phase transition with cluster formation in the system macro particle. The size of a cell depend on concentration macro particles. The coefficients $l_{lc}^2 \simeq \frac{1}{\xi^2}\frac{A}{r_0^2}\exp(-\xi r_0)$ in the free energy make the possibility determined the wave lengths the new structure. The criterion of instability given by this relation can be interpreted as a condition for formation of spatially non-uniform distribution at a given temperature, which depends on the concentration of macro particles and characteristic length of the new structure. In our case the result analytical calculation hive: $R_{lc} \simeq \sqrt{\frac{3T_n}{T_n - T}}\xi^{-1} \simeq \sqrt{\frac{K_n}{WnS}}$. When the $n \to 0$, $R \to \infty$, but $m \to 0$, and we have the clear nematic phase without the disclination. For particles by the size $3\,\mu$km at meanings $W = 10^{-5}\,\mathrm{J/m^2}$, $K = 10^{-11}\,\mathrm{J/m}$ and the concentration $c = 0{,}01$ the size of a cell has the order $20\,\mu$km and decreases at increase of concentration.

VIII. CONCLUSION

In this paper the potential of interaction is derived for the particles doped in nematic liquid crystal with taking into account of the collective screening effects, which are essential for the real colloid systems. Exponentially screened attraction of the magnetic grains in ferronematic explains "cellular" structure in the presence of the external magnetic field. Collective effects in doped nematic liquid crystals strongly depend on the anchoring strength, concentration and external fields. Collective action of all anisotropic particles leads to the exponential screening of the interaction.

The equilibrium distribution of spherical particles corresponds to their spatially non-uniform distribution. This spatial–non-uniform distribution of the introduced particles causes occurrence of areas with a plenty of defects. The mutual influence of particles and liquid crystal results in non-uniform distribution of cooperating particles and defects in a liquid crystal. Is formed some kind of a new soft body, which properties differ from properties of environment, separate a component. A wide variety of crystal structures is possible, if the forces of interaction possess anisotropy. Such interaction can also be realized in the system of spherical macro particles, cooperating via deformation of elastic field of the director in a liquid crystal. Another important property of such systems is that they are highly sensitive to weak external influences. It enables us to exert influence upon formation of the structures and their transmutations. Therefore, the macro particles introduced into liquid crystals, represent a system for research

of conditions of formation of possible crystalline structures. In addition, this system is advantageous that they are visually observable and one can directly observe all changes of the structures.

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